Tropospheric Delay Raytracing
Applied in VLBI Analysis

David Eriksson
NVI Inc./Chalmers Univ. of Technology

Dan MacMillan and John Gipson
NVI Inc.
Overview

1. Background
2. Raytrace Delay Computation
3. VLBI CONT11 Solutions Using Raytrace Delays
4. Raytrace Delay Results for Intensives
5. Conclusions
Elevation-dependent Tropospheric Delay

\[ \tau^{\text{symmetric}}_{\text{total}}(el) = m_{\text{hydrostatic}}(el)\tau^{\text{zenith}}_{\text{dry}} + m_{\text{wet}}(el)\tau^{\text{zenith}}_{\text{wet}} \]

Azimuthal-dependence approximated with Linear gradient model ("tilted atmosphere")

\[ \tau^{\text{gradient}}(el, az) = m_{\text{grad}}(el)[G_N \cos(az) + G_E \sin(az)] \]

\[ m_{\text{grad}}(el) = 1/(\sin(el) \tan(el) + C) \]
Current Troposphere Delay Model

NMF: (Niell, 1996)
- 1-dim raytrace of N Hemisphere radiosonde troposphere profile data
- Parametrized by day of year (annual period), latitude, and site height

VMF1: (Boehm et al., 2006)
- 1-dim raytrace of ECMWF tropospheric profile data
- Given at 6-hour intervals
- Spatially interpolated to each geodetic site
- Assumed that there is no horizontal refractivity variation

- Mapping functions m(el) were derived by raytracing through uniform atmospheric layers of constant refractivity
- Refractivity profile computed using the (Pressure, Temperature, Relative humidity) profile above the geodetic site location
Raytracing Approach

- Compute total (dry+wet) delays and wet mapping function from numerical weather model for each VLBI observation

- Weather model is the NASA/GSFC GEOS 5.9.1
  - parameters: pressure, temperature, specific humidity, geopotential height
  - time resolution: 3 hours
  - horizontal resolution: 0.5° x 0.625° (~ 50 km)
  - vertical resolution: 72 levels

- Refractivity along raypath is determined by interpolation of the 4D refractivity field

- Use piecewise linear approach to compute raytraced delays

- Constrain propagation of the ray to a plane of constant azimuth (to minimize computation time)
Observed/Raytraced Wet Zenith Delay

- NMF hydrostatic delay = a priori tropospheric delay
- Estimate wet zenith delay from VLBI data

- Average correlation all over all CONT11 sites = 0.93
- Raytraced delay accounts for ~90% of the observed delay
Validation Using VLBI Data

- VLBI data sets
  - CONT11
  - UT1 Intensives

- Compare troposphere delay models:
  - NMF hydrostatic delay + NMF wet mapping function
  - VMF1 total (dry+wet) delays + VMF1 wet mapping function
  - Raytrace total (dry+wet) delays + wet raytrace mapping function

- Estimated parameters: site positions, clocks, wet zenith, gradients

- Observation weighting options
  - Baseline weighting
  - Elevation dependent weighting
  - Correlated noise
Validation Using VLBI Data

• **Baseline weighting**
  Add a baseline-dependent noise to the formal observation uncertainty
  \( \Rightarrow \) chisquare/dof = 1

  \[ \sigma'_{12}^2 = \sigma_{12}^2 + \epsilon_{12}^2 \]

• **Elevation dependent weighting**
  Add an elevation-dependent noise

  \[ \sigma'_{12}^2 = \sigma_{12}^2 + [\epsilon_1 m(e1) + \epsilon_2 m(e2)]^2 \]

• **Correlated Noise**
  Second baseline from station 1

  \[ \sigma'_{13}^2 = \sigma_{13}^2 + [\epsilon_1 m(e1) + \epsilon_3 m(e2)]^2 \]

Observations are correlated \( \Rightarrow \) correlated noise term in the off-diagonal element of the covariance matrix between observations
## Validation Using VLBI Data

### CONT11 Baseline Length WRMS

<table>
<thead>
<tr>
<th>Weighting</th>
<th>NMF</th>
<th>VMF1 Total</th>
<th>Raytrace</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weighting Average (mm)</td>
<td>6.89</td>
<td>6.75</td>
<td>6.41</td>
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<tr>
<td>Baseline</td>
<td>6.50</td>
<td>6.31</td>
<td>6.04</td>
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<tr>
<td>Correlated noise</td>
<td>6.35</td>
<td>5.96</td>
<td>5.73</td>
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</table>
CONT11 Baseline Lengths

Improvement Relative to VMF1

• Ordered by baseline length for each site
Elevation cutoff test: Difference $5^\circ$ and $12^\circ$ solutions
$\Rightarrow$ measure of atmosphere model error

Raytrace: 0.017 ppb   VMF1: 0.075 ppb   NMF: 0.061 ppb
Intensive UT1 Sessions

• Compute VLBI LOD at midpoint between each pair of daily UT1 values

• Interpolate IGS LOD to these midpoint epochs

WRMS difference (VLBI – GPS) LOD (μs/day)

<table>
<thead>
<tr>
<th></th>
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<th>VMF1</th>
<th>Raytrace</th>
<th>Numsess</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kokee-Wettzell</td>
<td>25.4</td>
<td>25.2</td>
<td>24.3</td>
<td>80</td>
</tr>
<tr>
<td>Tsukuba-Wettzell</td>
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<td>28.3</td>
<td>26.1</td>
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Summary

- Compared with VMF1, baseline length repeatabilities are improved with raytracing for 70% of baselines

- Site vertical repeatabilities are improved for 11 of 13 CONT11 sites

- Troposphere scale bias for raytrace solution = 0.017 ppb compared to 0.075 ppb for VMF1 and 0.061 for NMF

- Raytraced wet zenith delay accounts for 90% of the observed wet zenith delay estimated from the VLBI data

- Computation time for the raytraced delay for each observation is 1 msec

- Raytracing service is available that provides raytrace delays for all VLBI sessions since 2000 at http://lacerta.gsfc.nasa.gov.tropodelays
Raytrace vs. Mapping Function

Path of refracted signal in troposphere

Path of signal in vacuum

\[ \tau_{atmos} = \int_{atmos} n(r)ds - \int_{vacuum} ds \]

Local vertical

Hydrostatic zenith delay: 2.3 m
Wet zenith delay: 5-50 cm
Tropospheric Delay at 5 deg: 25 m
Geometric excess at 5 deg contribution: 20 mm
Raytraced hydrostatic zenith (hydrostatic) and wet zenith delays at one epoch (2011-Sept-24-12UT)
2011-2013 Experiment Sessions
## Validation Using VLBI Data

### 2011-2013 Baseline Length WRMS

<table>
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<th>VMF1 Total</th>
<th>Raytrace</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weighting</td>
<td>Average (mm)</td>
<td>Average (mm)</td>
<td>Average (mm)</td>
</tr>
<tr>
<td>Baseline</td>
<td>10.76</td>
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<td>9.93</td>
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<tr>
<td>Elevation-dep</td>
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<td>10.34</td>
<td>10.13</td>
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<tr>
<td>Correlated noise</td>
<td>10.78</td>
<td>10.35</td>
<td>10.12</td>
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</table>
2011-2013 Experiment Sessions

Vertical component
Ray Tracing Better for: 20 stations
VMF1 Total Better for: 8 stations

North-South component
Ray Tracing Better for: 20 stations
VMF1 Total Better for: 6 stations

East-West component
Ray Tracing Better for: 17 stations
VMF1 Total Better for: 10 stations