Using a Kalman filter to regularize the VLBI nutation time series

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Contents

• Goal: State-of-the-art: VLBI nutation time series.
  – Importance of the VLBI nutation time series;
  – Why regularizing them to daily time series?
• A Kalman filter to regularize the VLBI nutation time series: nutkal2012.f.
  – How to use it? Control file and options.
• Applications:
  – Optimal choice of appropriate parameters;
  – Estimator of the quality of the estimate: the Goodness Of Fit.
• Conclusions and perspectives:
  – nutkal2012.f for prediction.
What is Nutation, anyway? (nutatio)
Why Regularize?

• Nutation estimated by VLBI at irregular intervals.
• Time-tag depends on epoch of the experiment.
• For many purposes nice to have regularly spaced data.
• We (IVS) are closest to the data. We should be able to produce the best results.
Our Approach

25 years ago we (GSFC) faced a similar problem for UT1 and PM. Needed a good a-priori ERP model. Developed eopkal (EOP Kalman filter) still in use.

10 years later developed nutkal.

This report is about nutkal2012. (An updated version of nutkal.)
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**Kalman filtering - Notions (1)**

Prior knowledge

\[
\hat{x}_{K-1|K-1}, \quad \hat{P}_{K-1|K-1}
\]

Prediction

\[
\hat{x}_{K|K-1}, \quad \hat{P}_{K|K-1}
\]

Add measurement

\[
Z, \quad P_Z
\]

New prediction

\[
\hat{x}_{K|K}, \quad \hat{P}_{K|K}
\]
Kalman filtering - Updating

• Prediction is linear:

\[ X_{K|K-1} = AX_{K-1|K-1} \]

\[ P_{K|K-1} = AP_{K-1|K-1}A^T + noise \]

The A embodies the physics.
Kalman filtering – Updating Example

• Example: *Linear motion.*
  
  – State: \[
  \begin{pmatrix}
  X_{K-1} \\
  V_{K-1}
  \end{pmatrix}
  \quad P = \begin{pmatrix}
  \sigma_x^2 & C_{XV} \\
  C_X & \sigma_V^2
  \end{pmatrix}
  \quad A = \begin{pmatrix}
  1 & \Delta t \\
  0 & 1
  \end{pmatrix}
  \]

  – Update

  \[
  \begin{pmatrix}
  X_{K-1} \\
  V_{K-1}
  \end{pmatrix}
  \rightarrow
  \begin{pmatrix}
  X_{K-1} + \Delta t V_{K-1} \\
  V_{K-1}
  \end{pmatrix}
  \]

  \[
P \rightarrow P = \begin{pmatrix}
  \sigma_x^2 + 2C_{XV}\Delta t + \Delta t^2\sigma_V^2 & C_{XV} + \Delta t V \\
  C_{XV} + \Delta t V & \sigma_V^2
  \end{pmatrix}
  \]
Kalman filtering – Adding measurement

- Add measurement: Schematically
  \[ X_{K|K} = \frac{P^{-1}_{K|K-1}X_{K|K-1} + P^{-1}_Z}{P^{-1}_{K|K-1} + P^{-1}_Z} \]

Note that \( P^{-1} \approx \frac{1}{\sigma^2} \): we are just combining according to sigmas.

- Let: \( X_{K|K-1} = \left( \frac{P^{-1}_{K|K-1} + P^{-1}_Z}{P^{-1}_{K|K-1} + P^{-1}_Z} \right) X_{K|K-1} + \left( \frac{P^{-1}_{K|K-1} X_{K|K-1} + P^{-1}_Z}{P^{-1}_{K|K-1} + P^{-1}_Z} \right) \)

- Note:
  - As \( P^{-1}_{K|K-1} \rightarrow \infty, P^{-1}_{K|K-1} \rightarrow 0 \) give more importance to data.
  - As \( P^{-1}_{K|K-1} \rightarrow 0, P^{-1}_{K|K-1} \rightarrow \infty \) give more importance to prediction.
Kalman filtering - Notions (5)

- Our model for nutation is: **Integrated Random walk + N harmonic terms.**
  Each term has associated with it some noise.
  The harmonics also have associated a width or Q factor.
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A Kalman filter to regularize VLBI nutation time series: nutkal2012.f

• The program reads in a series of nutation values from the snoop nutation files...

```fortran
! Nutkal2012 Control file,
! Lines that begin with "!" are comments
! If a ! appears in a line the rest of the line is ignored.
!
! Following control where the data comes from and how much is read in.
Input snoop.nut  !input file
Output testVH20b !output file
Data_Start 50000,0 !Read in data after this epoch (MJD)
Data_End 0000,00 ! Not required. If missing or 0,00 will read in all data.
Max_Psi_sig 1,0 !Discard data if the sigma is larger than this.
Max_Eps_sig 1,0
!
! Following controls where the filter is put and the span.
Filter_before 10,0 ! Start filter this much before the first data point
Filter_after 180,0 ! run the filter for This many days after end of data,
Filter_spacing 1,0 !Spacing of output points.
!
! Can also specify start and end times explicit:
Filter_start date
Filter_end date
!
! The following controls how the signal is modeled.
! The following control how the signal is modeled.
Linear YES/NO Process_noise
Linear YES 0,01
!
Harmonic Period_days Width_days Process_noise
Harmonic 450 0 0,01
Harmonic 510 0 0,01
Harmonic 385 0 0,01
```

• Control file.
Optimal choice for the Kalman filter parameters (1)

- Snoop file: GSC 2011a solution.
- Which periodic signals?
  - From the FCN (Lambert XX) = -430.21 days;
  - From our study in 2012 (IVS GM) R1 and R4 sessions weekly series = 450, 510 and 385 days;
  - From PSD = 470.25 days.
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**Optimal choice for the Kalman filter parameters (2)**

<table>
<thead>
<tr>
<th>Linear</th>
<th>Harmonic</th>
<th>Goodness Of Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Yes/No</td>
</tr>
<tr>
<td>Test 1</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Test 2</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Test 3</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Test 4</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Test 5</td>
<td>Yes</td>
<td>0.01</td>
</tr>
</tbody>
</table>

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**Optimal choice for the Kalman filter parameters (3)**

*.prn* files: observations and estimated values at the date of the observations.
Using a Kalman filter to regularize the VLBI nutation time series

**Optimal choice for the Kalman filter parameters (4)**

Snoop file and nutkal2012 daily series.
Using a Kalman filter to regularize the VLBI nutation time series

Problem: Filter runs away

*nutkal2012.f* for prediction

![Graph showing longitude and obliquity angles](image-url)
Conclusions and perspectives

Goddard is developing a nutation Kalman filter. Goal is to use the Kalman filter to regularize the nutation series.

This series can be used:

- As an a priori model for VLBI data analysis.
- For geophysical investigations.

Still a work in progress.