

# An Experimental Kalman Filter Approach to the International Terrestrial Reference Frame Realization

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# Current ITRF Status and Rationale

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- **Secular frame characterized by  $X$ ,  $V$  and a full covariance matrix**
  - Linear motion models for all sites
  - Origin at mean but not instantaneous CM
    - CM for secular motion
    - Close to CF for sub-secular motions
  - The linear motion model works very well and the ITRF2008 frame is quite stable at 0.3 – 0.5 mm/yr
- **The case for an Experimental Kalman Filter Approach**
  - Non-linear motion, sites with short data span
  - Near real-time orbit determination and global monitoring
  - Unify different geodetic data time series in one and the same frame
  - Origin at nearly instantaneous CM



# Experimental Kalman Filter Approach to TRF

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- **One reference frame** realized by time series
- **Origin defined at weekly (nearly instantaneous) CM**
  - Currently through SLR data
  - Could take other data or models in the future
- **Scale realized by weekly SLR/VLBI data**
- **Orientation defined weekly by convention and no net rotation**
- **Local ties are applied only once in the weeks of surveying or within the continuous segments without offsets**
- **Co-motion constraints are applied to most if not all co-located sites**



# Weekly Combination Strategy

- Use CATREF heritage
- Combination done at weekly basis
- Kalman Filter Data Update

- Coordinates in file k

$$X_s^i = X_c^i + (t_s^i - t_0) \dot{X}_c^i + T_k + D_k X_c^i + R_k X_c^i \\ + (t_s^i - t_k) [\dot{T}_k + \dot{D}_k X_c^i + \dot{R}_k X_c^i]$$

- Local Ties applied once

- Tight Orientation Constraints every week

$$\mathbf{0} = \mathbf{B}(\mathbf{X}_c - \mathbf{X}_r)$$

- Weekly displacements at most co-located sites are constrained to be the same

EOP in file k

$$x_s^p = x_c^p + R2_k$$

$$y_s^p = y_c^p + R1_k$$

$$UT_s = UT_c - \frac{1}{f} R3_k$$

$$\dot{x}_s^p = \dot{x}_c^p$$

$$\dot{y}_s^p = \dot{y}_c^p$$

$$LOD_s = LOD_c$$



# Time Update or Prediction

- **Coordinate Decomposition**

$$\mathbf{X}_c = \begin{pmatrix} X_c^1 \\ Y_c^1 \\ Z_c^1 \\ \cdot \\ X_c^i \\ \cdot \end{pmatrix} = \mathbf{X} + \mathbf{S}_p$$

- **Equation of Dynamics**

- **Time Update or State Transition**

$$\begin{bmatrix} X_k \\ V_k \\ S_k^{next} \\ S_k^{now} \end{bmatrix} = \begin{bmatrix} 1 & dt & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2e^{-dt/\tau} \cos 2\pi \frac{dt}{T} & -e^{-2dt/\tau} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_{k-1} \\ V_{k-1} \\ S_{k-1}^{next} \\ S_{k-1}^{now} \end{bmatrix} + \begin{bmatrix} \varepsilon_x \\ \varepsilon_v \\ \varepsilon_p \\ 0 \end{bmatrix}$$

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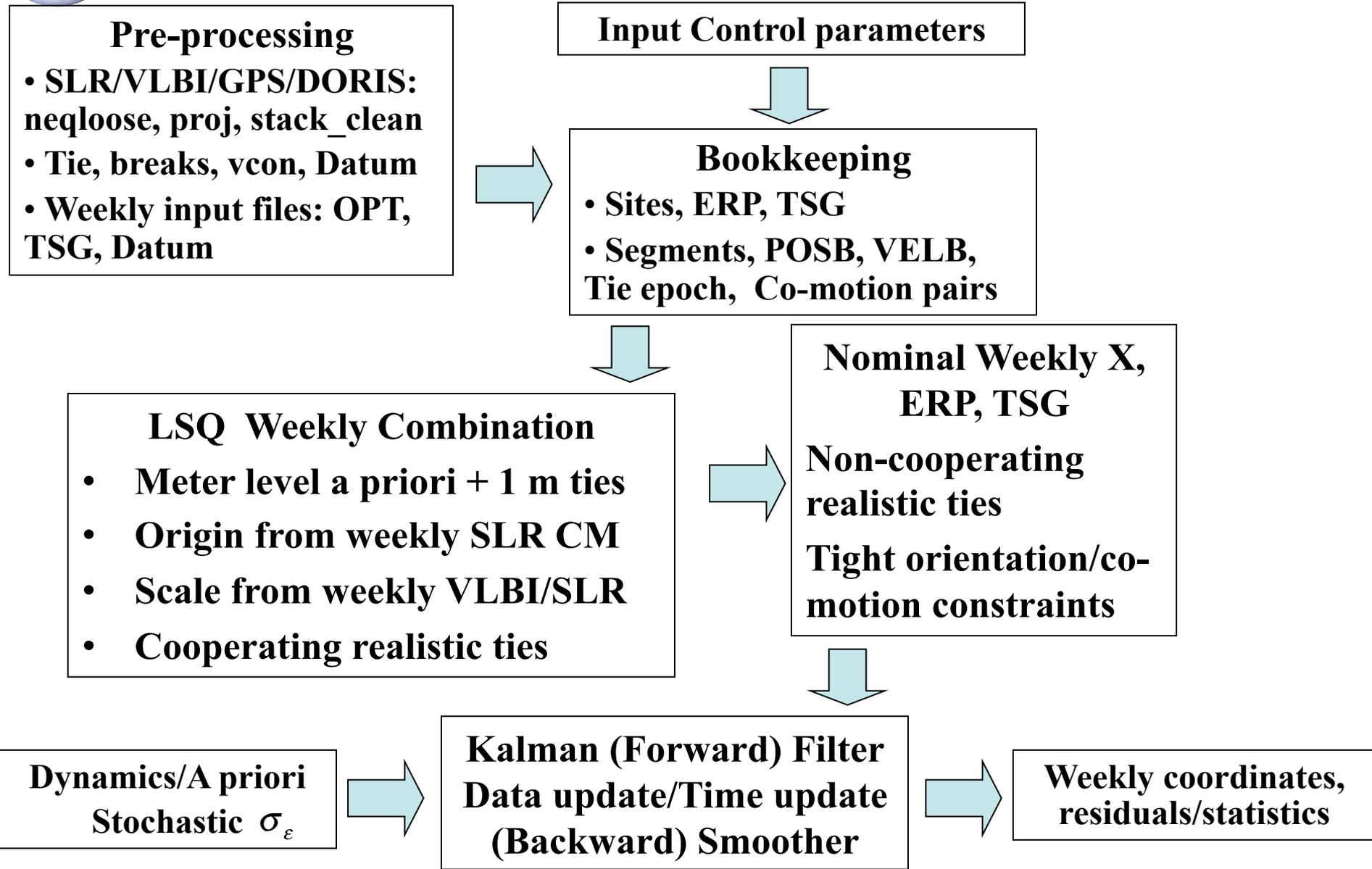
Abbondanza et al.

$$\mathbf{EOP}_k = \mathbf{EOP}_{k-1} + \varepsilon_{\mathbf{EOP}}$$

$$\mathbf{T}_k = \mathbf{T}_{k-1} + \varepsilon_{\mathbf{T}}$$

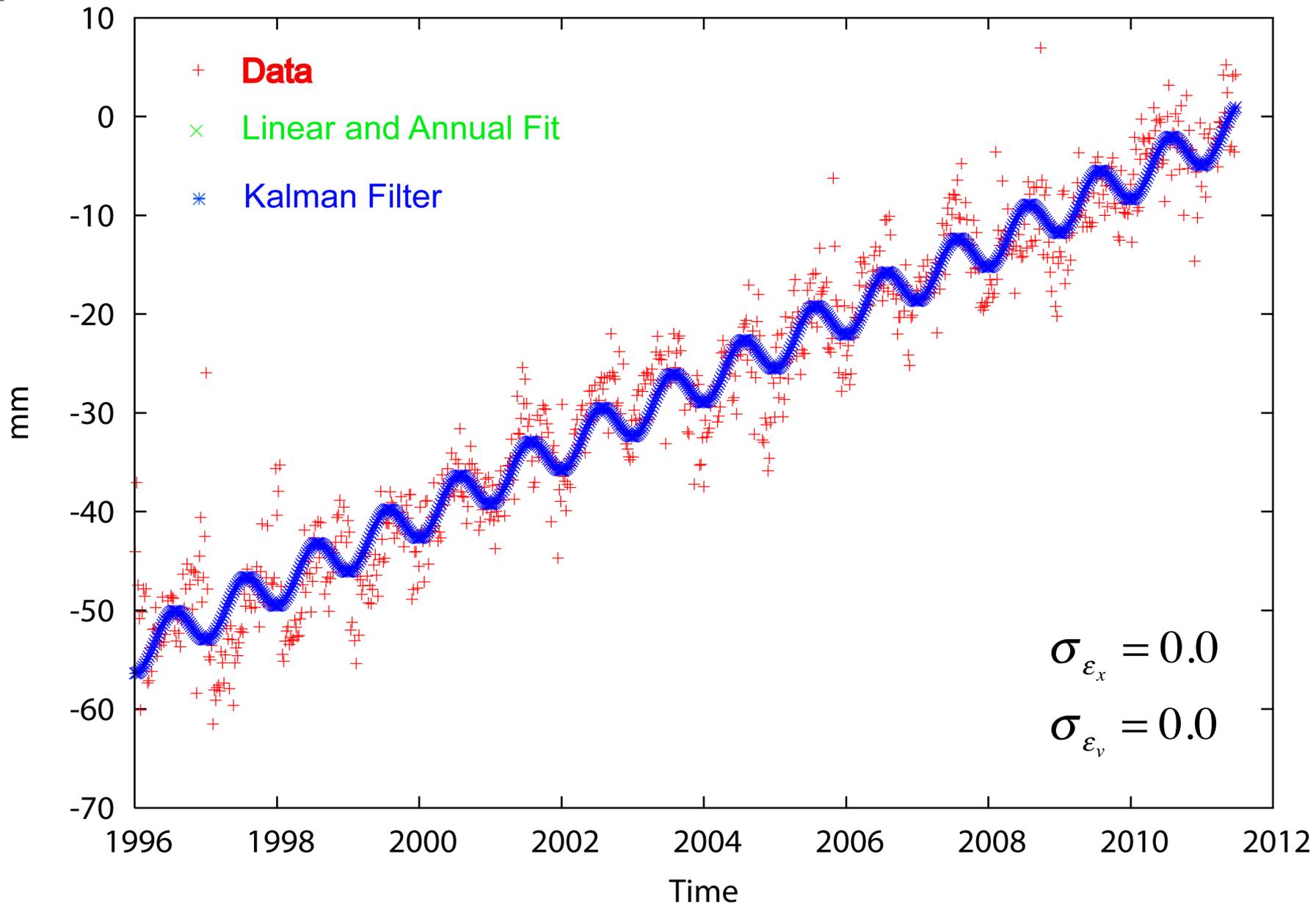


# Flow Chart of Weekly Combination Filter-Smoother



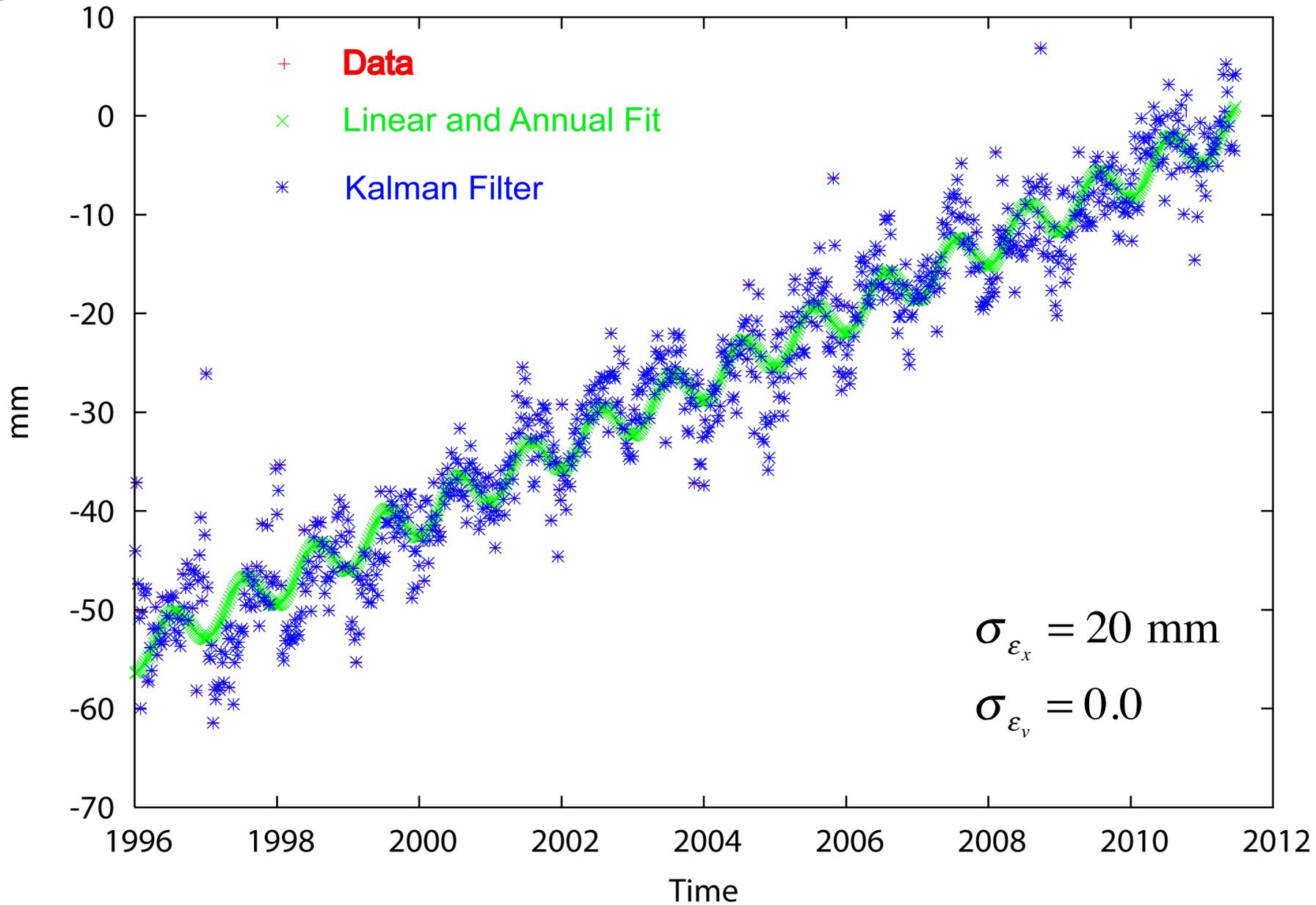


# Kalman Filter and RTS Smoother



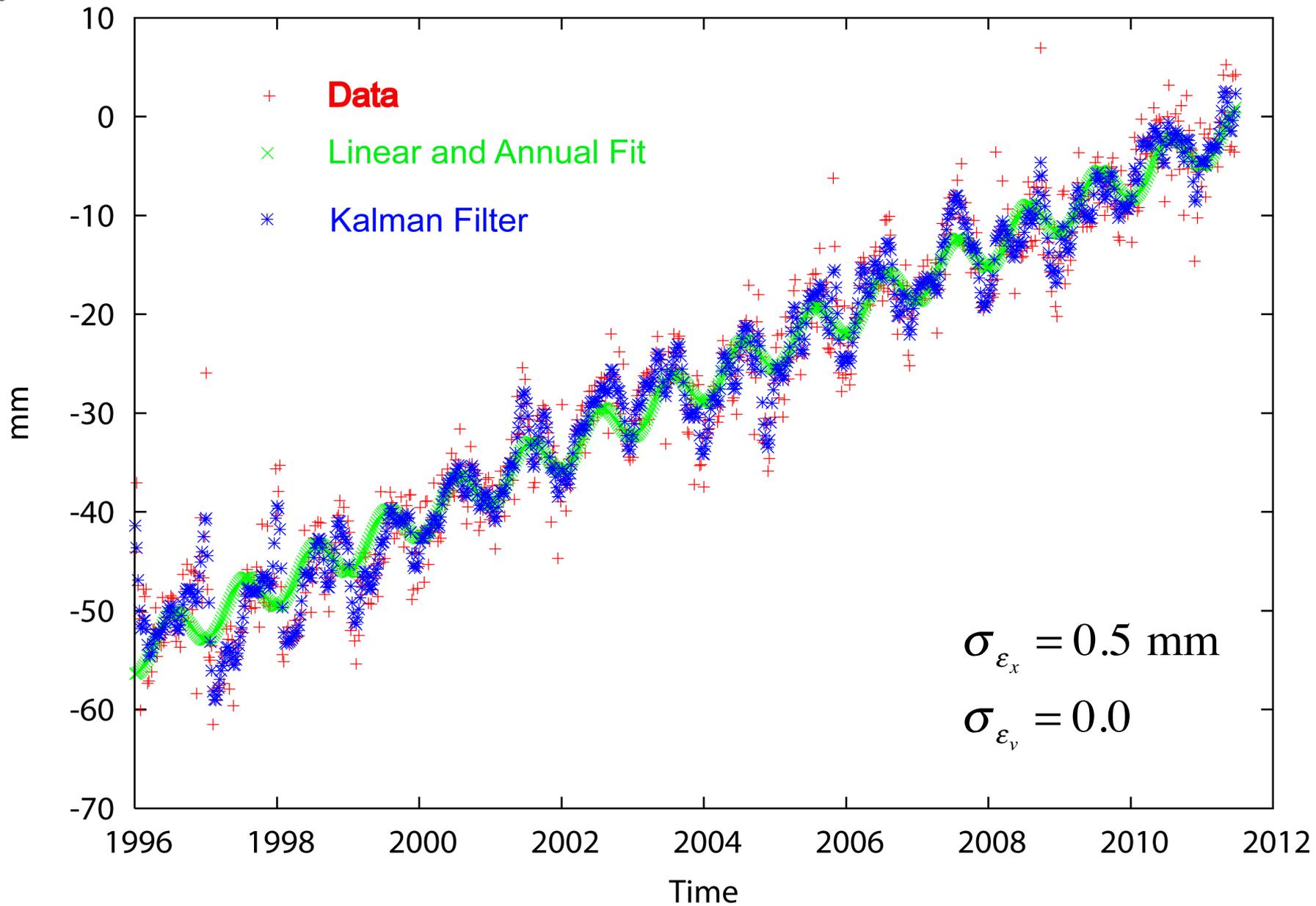


# Kalman Filter and RTS Smoother





# Kalman Filter and RTS Smoother





# Transformation Parameters between ITRF2005 and Weekly Combination (1996-2005)

## No Annual Components

Tx (mm)	Ty (mm)	Tz (mm)	D (ppb)	Rx ( $\mu$ as)	Ry ( $\mu$ as)	Rz ( $\mu$ as)
0.1	-0.1	-0.4	-0.69	-26	-1	11
Vx (mm/y)	Vy (mm/y)	Vz (mm/y)	(ppb/y)	$\dot{R}_x$ ( $\mu$ as/y)	$\dot{R}_y$ ( $\mu$ as/y)	$\dot{R}_z$ ( $\mu$ as/y)
0.0	0.0	0.3	0.00	10	-5	3

## With Annual Components

Tx (mm)	Ty (mm)	Tz (mm)	D (ppb)	Rx ( $\mu$ as)	Ry ( $\mu$ as)	Rz ( $\mu$ as)
0.1	0.1	-0.4	-0.65	-26	-2	11
Vx (mm/y)	Vy (mm/y)	Vz (mm/y)	(ppb/y)	$\dot{R}_x$ ( $\mu$ as/y)	$\dot{R}_y$ ( $\mu$ as/y)	$\dot{R}_z$ ( $\mu$ as/y)
0.0	0.0	0.2	-0.01	10	-5	3



# Summary

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- Consistently and accurately defined and realized TRF is essential for global change monitoring
- Experimental TRF realized by nearly instantaneous geocentric time series and combinations are done weekly
- The Kalman filter and RTS smoother offer great power and flexibility to estimate time-dependent parameters. Easy on constraints over variables such as displacements
- Fragmented time series at co-located sites are unified dynamically
- Time series from different geodetic techniques are unified in the same frame