KALREF – A Kalman Filter Approach to the International Terrestrial Reference Frame Realization

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Summary

- **A geocentric reference frame realized by time series**
- **Origin defined at nearly-instantaneous (weekly or daily) CM**
  - Currently through SLR data
  - Could take other data or models in the future
- **Scale realized by nearly-instantaneous SLR/VLBI data**
- **Geocentric coordinate time series to monitoring linear/non-linear motion instead of linear coordinate and velocity models**
- **Bridge fragmented geodetic observations at co-located stations**
- **A platform to unify time series of various geodetic techniques**

Introduction

1. Current ITRF Status:

Secular frame characterized by X, V and a full covariance matrix from SLR, VLBI, GPS and DORIS

- Piece-wise linear motion models for all sites
- Origin at mean but not instantaneous CM
  - CM for secular motion
  - Close to CF for sub-secular motions
- The linear motion model works well and the ITRF2008 frame is quite stable at 0.3 – 0.5 mm/yr
- Geodetic techniques have various strengths in frame parameters
- Difficult to transfer nearly instantaneous scale information among geodetic techniques with the linear model

2. Motivation:

- Geodetic sites are moving constantly and non-linearly
- Co-located stations of different techniques with fragmented and short time spans
- Need nearly-instantaneous geocentric (CM) coordinates (with scale) for real-time orbit determination and global change monitoring
- Geodetic time series of different techniques are in various frames

Methodology

1. Key Features:

- **Origin defined and realized at nearly instantaneous CM**
- **Scale realized by weekly or daily SLR/VLBI data**
- **Orientation defined nearly-instantaneously by convention and the no net rotation condition**
- **Local ties are applied once in the weeks of surveying or within the continuous segments without offsets**
- **Co-motion constraints are applied to most co-located sites**
- **Kalman Filter/RTS Smoother with fixed weekly time step**
- **Use Jean Boy's geophysical loading model for process noise variances (http://loading.u-strasbg.fr/)**

2. Combination Strategy

- Use CATREF/ITRF Heritage from IGN
- Filter/Smoother Codes from JPL
- Combination done at weekly basis with all techniques using weekly/daily files

Coordinates in file k

$$\mathbf{X}^i_s = \mathbf{X}^i_c + \mathbf{T}_k + \mathbf{D}_k\mathbf{X}^i_c + \mathbf{R}_k\mathbf{X}^i_c$$

EOP in file k

$$x^p_s = x^p_c + R2_k$$
$$y^p_s = y^p_c + R1_k$$
$$UT_s = UT_c - \frac{1}{f}R3_k$$
$$\dot{x}^p_s = \dot{x}^p_c$$
$$\dot{y}^p_s = \dot{y}^p_c$$
$$LOD_s = LOD_c$$

3. Kalman Filter Data Update

Filter Data Types:

- Weekly combined position $\mathbf{X}_c$, EOP $\mathbf{X}_p$, Transformation parameters $\mathbf{T}_k$, $\mathbf{D}_k$, $\mathbf{R}_k$ for file k
- Local Ties
- Tight weekly orientation constraints; Constraints for most co-located stations to move together using pseudo-data forms with uncertainties in 0.1 mm or 0.1 mm/yr shown in the measurement equation below:

$$0 = \mathbf{B}(\mathbf{X}_c - \mathbf{X}_r)$$

Methodology continued ↗

Methodology (Continued)

4. Kalman Filter Time Update:

- **Geocentric Coordinate Decomposition:** $\mathbf{X}_c = (\ x^1_c \quad y^1_c \quad z^1_c \quad . \quad x^i_c \quad .\ )^{\mathrm{T}} = \mathbf{X} + \mathbf{S}_{\mathbf{now}}$
- **Equation of Dynamics**
- **Time Update or State Transition:**

$$\begin{bmatrix} X_k \\ V_k \\ S^{next}_k \\ S^{now}_k \end{bmatrix} = \begin{bmatrix} 1 & dt & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2e^{-dt/\tau}\cos 2\pi\frac{dt}{T} & -e^{-2dt/\tau} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_{k-1} \\ V_{k-1} \\ S^{next}_{k-1} \\ S^{now}_{k-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_x \\ \varepsilon_v \\ \varepsilon_p \\ 0 \end{bmatrix}$$

$$\mathbf{EOP_k} = \mathbf{EOP_{k\text{-}1}} + \boldsymbol{\varepsilon}_{\mathbf{EOP}}$$

$$(T_k, D_k, R_k)^{\mathrm{T}} \equiv \mathbf{T_k} = \mathbf{T_{k\text{-}1}} + \boldsymbol{\varepsilon}_{\mathbf{T}}$$

$\varepsilon_x, \varepsilon_v, \varepsilon_p, \boldsymbol{\varepsilon}_{\mathbf{EOP}}, \boldsymbol{\varepsilon}_{\mathbf{T}}$ are process noises

$\varepsilon_v = 0,\ \varepsilon_p = 0$

$\varepsilon_x \sim N(0,\ \mathrm{Q}_x)$

- **Co-motion Constraints Are Applied in Two Different Ways:**
  - Velocities, S_{now}, S_{next} (of each frequency) of co-located stations constrained in the first week
  - Process noises of co-located stations are constrained to be the same through the Q_x matrix
- **Position or velocity breaks are realized through large process noise updates ε_x or ε_v**

5. Kalman Filter and RTS Smoother

State Transition: $\mathbf{x}_{k+1} = \mathbf{\Phi}_k\mathbf{x}_k + \mathbf{\Gamma}_k\boldsymbol{\varepsilon}_k,\ \ \boldsymbol{\varepsilon}_k \sim N(0, \mathbf{Q}_k)$

Data: $\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{\Delta}_k,\ \ \mathbf{\Delta}_k \sim N(0, \mathbf{R}_k)$

Filter

Gain: $\mathbf{K}_{fk} = \mathbf{P}^-_{fk}\mathbf{H}^T_k[\mathbf{H}_k\mathbf{P}^-_{fk}\mathbf{H}^T_k + \mathbf{R}_k]^{-1}$

Data Update: $\hat{\mathbf{x}}^+_{fk} = \hat{\mathbf{x}}^-_{fk} + \mathbf{K}_{fk}[\mathbf{y}_k - \mathbf{H}_k\hat{\mathbf{x}}^-_{fk}],\ \ \mathbf{P}^+_{fk} = [\mathbf{I} - \mathbf{K}_{fk}\mathbf{H}_k]\mathbf{P}^-_{fk}$

Time Update: $\hat{\mathbf{x}}^-_{fk+1} = \mathbf{\Phi}_k\mathbf{x}^+_{fk},\ \ \mathbf{P}^-_{fk+1} = \mathbf{\Phi}_k\mathbf{P}^+_{fk}\mathbf{\Phi}^T_k + \mathbf{\Gamma}_k\mathbf{Q}_k\mathbf{\Gamma}^T_k$

Smoother

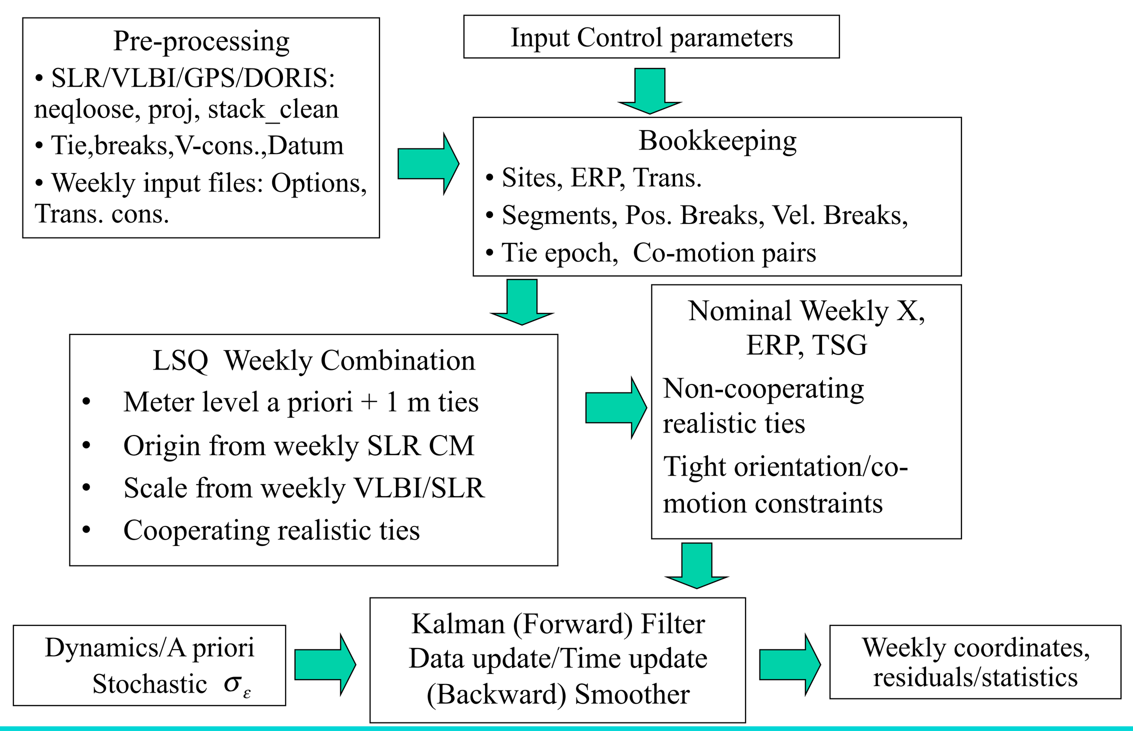
Initialize: $\hat{\mathbf{x}}_N = \hat{\mathbf{x}}^+_{fN},\ \ \mathbf{P}_N = \mathbf{P}^+_{fN}$

Gain: $K_k = \mathbf{P}^+_{fk}\mathbf{\Phi}^T_k(\mathbf{P}^-_{fk+1})^{-1}$

Estimates: $\hat{\mathbf{x}}_k = \mathbf{x}^+_{fk} + K_k[\hat{\mathbf{x}}_{k+1} - \hat{\mathbf{x}}^-_{fk+1}]$

Covariance: $\mathbf{P}_k = \mathbf{P}^+_{fk} - K_k[\mathbf{P}^-_{fk+1} - \mathbf{P}_{k+1}]K^T_k$

KALREF Flow Chart



Results

1. Transformation from ITRF to KALREF Solutions (No Stochastics)

ITRF2005 to KALREF (No Annual Components)

Tx (mm)	Ty (mm)	Tz (mm)	D (ppb)	Rx (µas)	Ry (µas)	Rz (µas)
0.1 ± 0.1	0.0 ± 0.1	0.4 ± 0.2	0.0 ± 0.01	1 ± 4	1 ± 4	-1 ± 3
Vx (mm/y)	Vy (mm/y)	Vz (mm/y)	$\dot{D}$ (ppb/y)	$\dot{R}_x$ (µas/y)	$\dot{R}_y$ (µas/y)	$\dot{R}_z$ (µas/y)
-0.0±0.04	0.06±0.03	0.03±0.07	0.0±0.003	-0.3 ± 0.6	2.6 ± 0.6	0.6 ± 0.3

ITRF2008 to KALREF (No Annual Components)

Tx (mm)	Ty (mm)	Tz (mm)	D (ppb)	Rx (µas)	Ry (µas)	Rz (µas)
0.2 ± 0.2	-0.4 ± 0.1	0.4 ± 0.2	0.43 ± 0.1	-16 ± 5	-1 ± 6	-12 ± 5
Vx (mm/y)	Vy (mm/y)	Vz (mm/y)	$\dot{D}$ (ppb/y)	$\dot{R}_x$ (µas/y)	$\dot{R}_y$ (µas/y)	$\dot{R}_z$ (µas/y)
0.1±0.03	-0.1±0.03	0.1±0.04	0.0±0.02	1 ± 1	2 ± 1	-5 ± 1

ITRF2005 to KALREF with Annual Components

Tx (mm)	Ty (mm)	Tz (mm)	D (ppb)	Rx (µas)	Ry (µas)	Rz (µas)
0.1 ± 0.1	0.1 ± 0.1	0.3 ± 0.3	0.0 ± 0.02	3 ± 4	1 ± 4	-1 ± 4
Vx (mm/y)	Vy (mm/y)	Vz (mm/y)	$\dot{D}$ (ppb/y)	$\dot{R}_x$ (µas/y)	$\dot{R}_y$ (µas/y)	$\dot{R}_z$ (µas/y)
-0.0±0.04	0.09±0.04	0.07±0.08	-0.0±0.01	0.1 ± 0.7	2.5 ± 0.7	0.5 ± 0.4

ITRF2008 to KALREF with Annual Components

Tx (mm)	Ty (mm)	Tz (mm)	D (ppb)	Rx (µas)	Ry (µas)	Rz (µas)
0.1 ± 0.2	-0.2 ± 0.1	0.2 ± 0.2	0.46 ± 0.1	-17 ± 6	1 ± 6	-13 ± 5
Vx (mm/y)	Vy (mm/y)	Vz (mm/y)	$\dot{D}$ (ppb/y)	$\dot{R}_x$ (µas/y)	$\dot{R}_y$ (µas/y)	$\dot{R}_z$ (µas/y)
0.05±0.04	-0.1±0.03	0.1±0.05	0.0±0.02	1.5 ± 0.8	2.8 ± 0.8	-5 ± 0.8

KALREF Gets Results Similar to ITRF under Linear Model

Results continued ↗

Results (continued)

2. Geocentric Coordinate Time Series

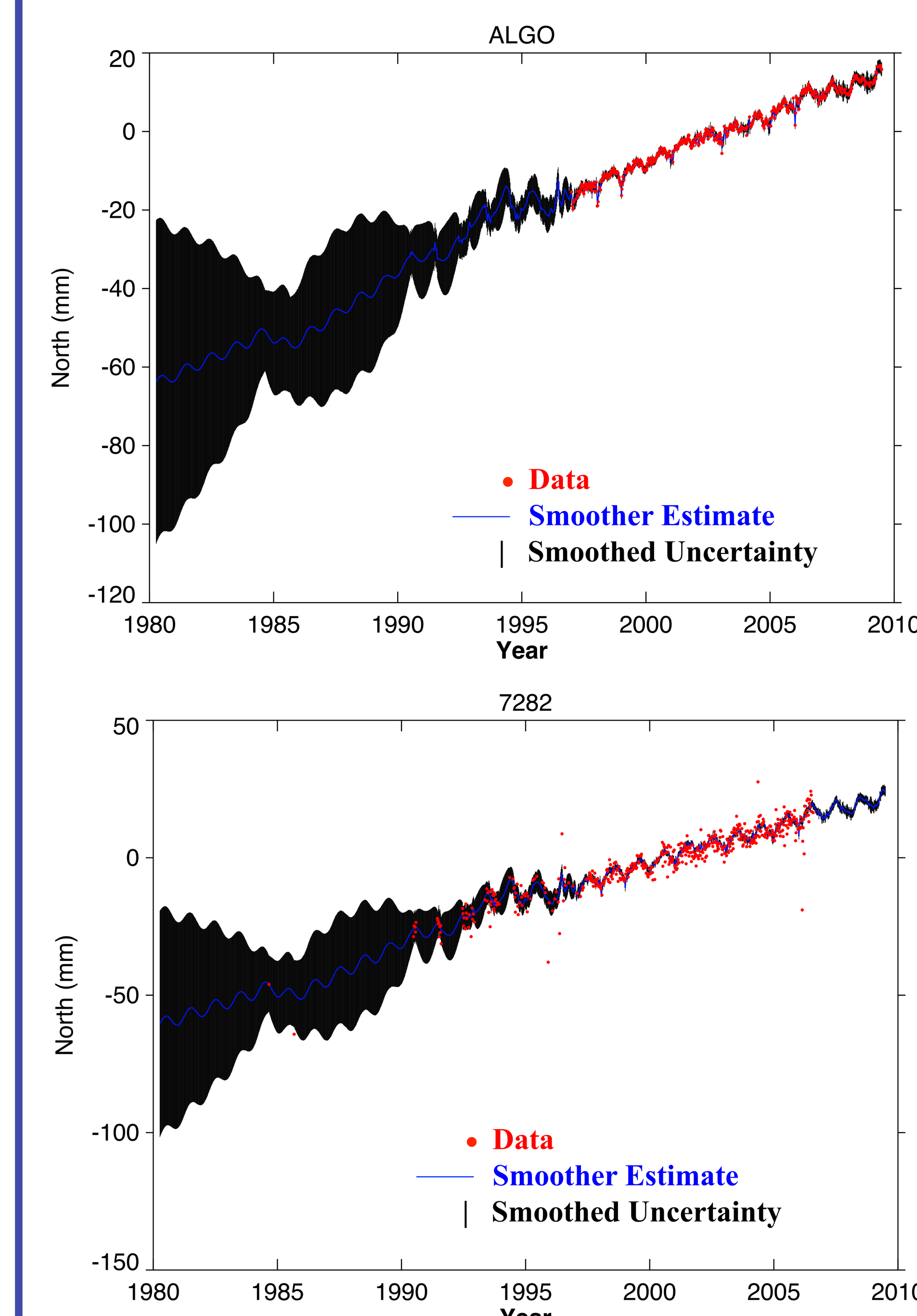


Figure 1. Time series of co-located GPS (Top: ALGO) and VLBI (Bottom: 7282) stations. The estimates benefit from data at co-located stations due to co-motion constraints. The site also has a co-located SLR station 7410 with a very short tracking history (not shown).

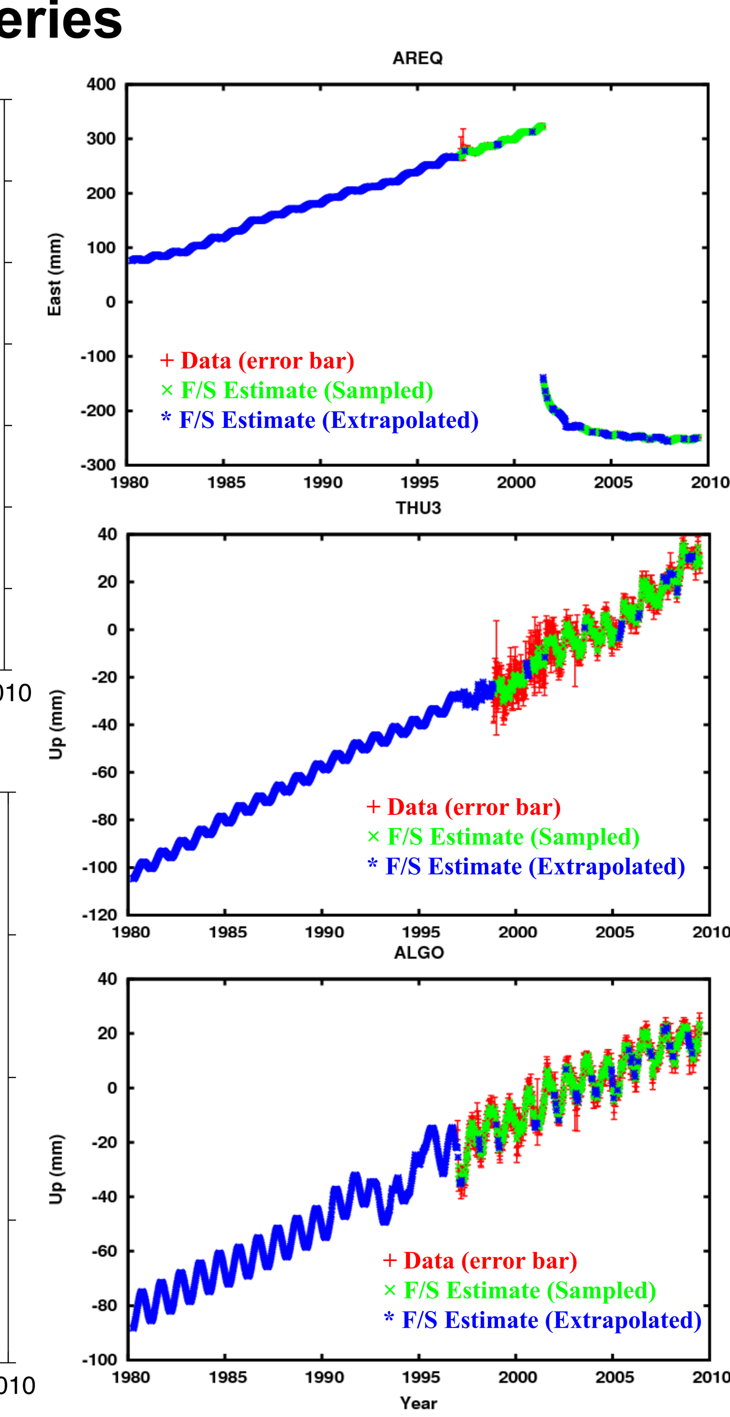


Figure 2. Time series of 3 sites. Top: GPS east component at AREQ covering the 2001 Arequipa Earthquake. Middle: GPS vertical at THU3 in Greenland showing ice loss acceleration (*Jiang et al., 2010*). Bottom: GPS vertical at ALGO.

3. KALREF Geocenter Motion

Table 3. Comparison of Annual Geocenter Motion Estimates

Data	X_g		Y_g		Z_g		Ref
	Amp mm	Phase day	Amp mm	Phase day	Amp mm	Phase day	
SLR (L1/L2)	2.2	60	3.2	303	2.8	46	*Eanes et al., 1997*
SLR (5 satellites)	3.2 ± 0.4	33 ± 3	2.6 ± 0.2	306 ± 2	4.3 ± 0.3	31 ± 2	*Cheng et al., 2002-2010*
GPS+OBP+GRACE	1.8 ± 0.1	49 ± 4	2.7 ± 0.1	329 ± 2	4.2 ± 0.2	31 ± 3	*Wu et al., 2002-2009*
KALREF 82 sites Filtered	1.5 ± 0.1	54 ± 2	1.9 ± 0.1	322 ± 1	3.5 ± 0.1	20 ± 1	*2002-2009*
KALREF 82 sites Not Filtered	2.0 ± 0.2	77 ± 5	2.5 ± 0.1	316 ± 3	3.6 ± 0.3	25 ± 5	*2002-2009*

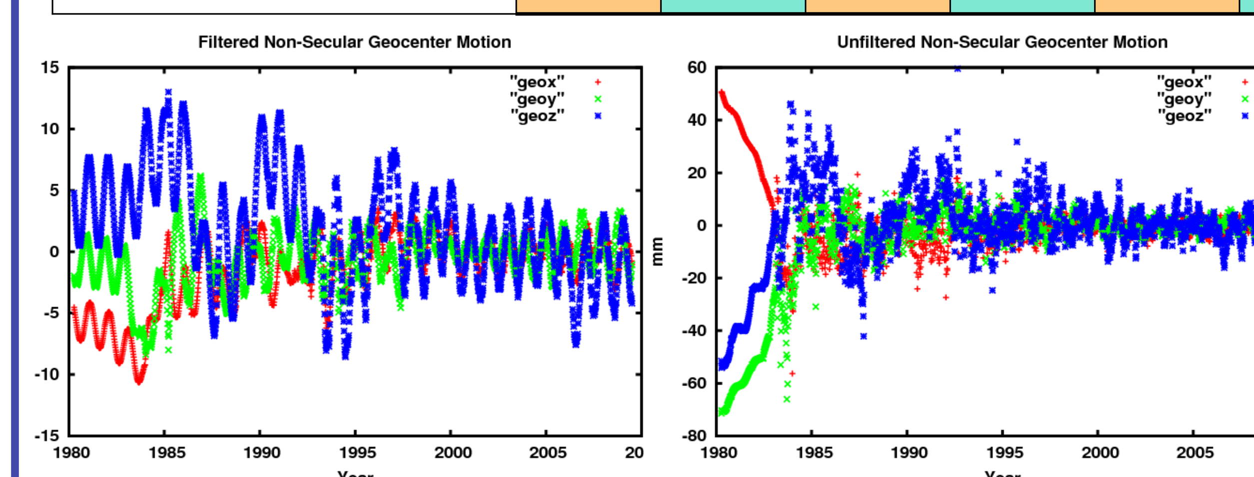


Figure 3. Geocenter motion time series from 82 global sites. Left: derived from smoothed coordinate series using the loading model. Right: derived from estimated series with 5 cm process noise. Such series follow the unfiltered raw data.

Conclusions

- Consistent and accurately defined and realized TRF is essential for global change monitoring
- Experimental TRF realized by nearly instantaneous geocentric time series and weekly combinations
- Kalman filter and RTS smoother offer power and flexibility for time-dependent parameters and constraints
- KALREF applied to ITRF2005/2008 input data and linear solutions yield good agreement with ITRF
- Unifies fragmented time series with co-locations from 4 geodetic techniques in the same geocentric frame